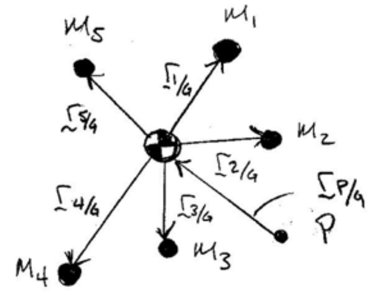
Parallel Axis Theorem



Let $\underset{\sim}{r}_{j/G} = x_{j/G}\hat{i} + y_{j/G}\hat{j}$

Then the moment of inertia about the center of mass is

$$I_G = \sum_{j=1}^{N} m_j\left(x_{j/G}^2 + y_{j/G}^2\right)$$

Note that $x_{j/G}^2 + y_{j/G}^2 = \underset{\sim}{r}_{j/G} \cdot \underset{\sim}{r}_{j/G}$

So $I_G = \sum_{j=1}^{N} m_j \, \underset{\sim}{r}_{j/G} \cdot \underset{\sim}{r}_{j/G}$

Now the moment of inertia about point P is:

$$I_P = \sum_{j=1}^{N} m_j \, \underset{\sim}{r}_{j/P} \cdot \underset{\sim}{r}_{j/P}$$

But

$$\underset{\sim}{r}_{j/P} \doteq \underset{\sim}{r}_{G/P} + \underset{\sim}{r}_{j/G}$$

So

$$I_P = \sum_{j=1}^{N} m_j \left(\underset{\sim}{r}_{G/P} + \underset{\sim}{r}_{j/G}\right) \cdot \left(\underset{\sim}{r}_{G/P} + \underset{\sim}{r}_{j/G}\right)$$

$$\Rightarrow I_P = \sum_{j=1}^{N} m_j \left(\underset{\sim}{r}_{G/P} \cdot \underset{\sim}{r}_{G/P} + 2\, r_{G/P} \cdot r_{j/G} + r_{j/G} \cdot r_{j/G} \right)$$

$$= \sum_{j=1}^{N} \left(m_j\, \underset{\sim}{r}_{G/P} \cdot \underset{\sim}{r}_{G/P} \right) + 2 \sum_{j=1}^{N} \left(r_{G/P} \cdot r_{j/G} \right) m_j + \sum_{j=1}^{N} m_j\, r_{j/G} \cdot r_{j/G}$$

$$I_P = \underset{\sim}{r}_{G/P} \cdot \underset{\sim}{r}_{G/P} \overbrace{\left(\sum m_j \right)}^{m} + 2\, \underset{\sim}{r}_{G/P} \cdot \underbrace{\sum_{j=1}^{N} m_j r_{j/G}}_{=0} + \underbrace{\sum_{j=1}^{N} m_j\, \underset{\sim}{r}_{j/G} \cdot \underset{\sim}{r}_{j/G}}_{I_G}$$

So

$$I_P = I_G + m\, \underset{\sim}{r}_{G/P} \cdot \underset{\sim}{r}_{G/P}$$

$$\boxed{I_P = I_G + m \left(x_{G/P}^2 + y_{G/P}^2 \right)}$$

Note $x_{G/P}^2 + y_{G/P}^2$ is the distance squared between points G & P